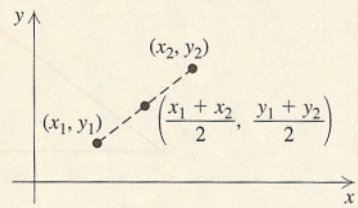
Student Notes

To help remember the formulas correctly, note that the distance formula (a variation on the Pythagorean theorem) involves both subtraction and addition, whereas the midpoint formula does not include any subtraction.

The Midpoint Formula If the endpoints of a segment are (x_1, y_1) and (x_2, y_2), then the coordinates of the midpoint are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



(To locate the midpoint, average the x-coordinates and average the y-coordinates.)

EXAMPLE 8 Find the midpoint of the segment with endpoints $(-2, 3)$ and $(4, -6)$.

SOLUTION Using the midpoint formula, we obtain

$$\left(\frac{-2 + 4}{2}, \frac{3 + (-6)}{2}\right), \text{ or } \left(\frac{2}{2}, \frac{-3}{2}\right), \text{ or } \left(1, -\frac{3}{2}\right).$$

Try Exercise 65.

7.7 Exercise Set

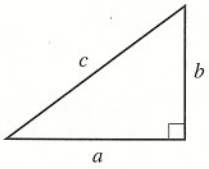
FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *Complete each sentence with the best choice from the column on the right.*

1. In any (d) triangle, the square of the length of the hypotenuse is the sum of the squares of the lengths of the legs.
2. The shortest side of a right triangle is always one of the two (c).
3. The principle of (e) states that if $x^2 = n$, then $x = \sqrt{n}$ or $x = -\sqrt{n}$.
4. In a(n) (b) right triangle, both legs have the same length.
5. In a(n) (f) right triangle, the hypotenuse is twice as long as the shorter leg.
6. If both legs in a right triangle have measure a, then the (a) measures $a\sqrt{2}$.

a) Hypotenuse
b) Isosceles
c) Legs
d) Right
e) Square roots
f) 30°–60°–90°

In a right triangle, find the length of the side not given. Give an exact answer and, where appropriate, an approximation to three decimal places.



7. $a = 5, b = 3$ $\sqrt{34}$; 5.831

8. $a = 8, b = 10$ $\sqrt{164}$; 12.806

Aha! **9.** $a = 9, b = 9$ $9\sqrt{2}$; 12.728

10. $a = 10, b = 10$ $10\sqrt{2}$; 14.142

11. $b = 12, c = 13$ 5

12. $a = 7, c = 25$ 24

In Exercises 13–28, give an exact answer and, where appropriate, an approximation to three decimal places.

13. A right triangle's hypotenuse is 8 m and one leg is $4\sqrt{3}$ m. Find the length of the other leg. 4 m

14. A right triangle's hypotenuse is 6 cm and one leg is $\sqrt{5}$ cm. Find the length of the other leg. $\sqrt{31}$ cm; 5.568 cm

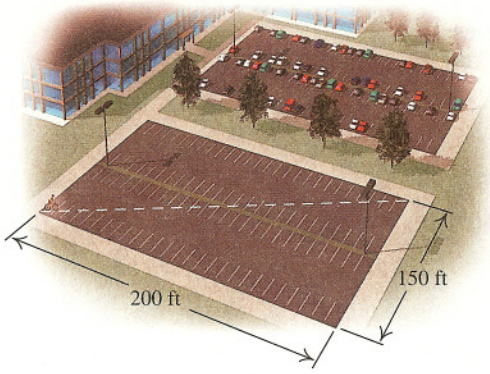
15. The hypotenuse of a right triangle is $\sqrt{20}$ in. and one leg measures 1 in. Find the length of the other leg. $\sqrt{19}$ in.; 4.359 in.

16. The hypotenuse of a right triangle is $\sqrt{15}$ ft and one leg measures 2 ft. Find the length of the other leg. $\sqrt{11}$ ft; 3.317 ft

Aha! **17.** One leg of a right triangle is 1 m and the hypotenuse measures $\sqrt{2}$ m. Find the length of the other leg. 1 m

18. One leg of a right triangle is 1 yd and the hypotenuse measures 2 yd. Find the length of the other leg. $\sqrt{3}$ yd; 1.732 yd

19. *Bicycling.* Amanda routinely bicycles across a rectangular parking lot on her way to class. If the lot is 200 ft long and 150 ft wide, how far does Amanda travel when she rides across the lot diagonally? 250 ft

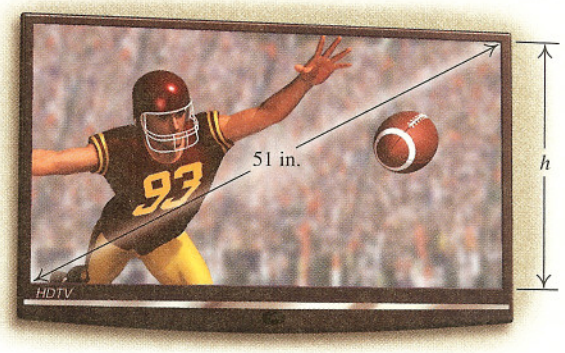


20. *Guy Wire.* How long is a guy wire if it reaches from the top of a 15-ft pole to a point on the ground 10 ft from the pole? $\sqrt{325}$ ft; 18.028 ft

21. *Softball.* A slow-pitch softball diamond is actually a square 65 ft on a side. How far is it from home plate to second base? $\sqrt{8450}$, or $65\sqrt{2}$ ft; 91.924 ft

22. *Baseball.* Suppose the catcher in Example 1 makes a throw to second base from the same location. How far is that throw? $\sqrt{14{,}500}$ ft; 120.416 ft

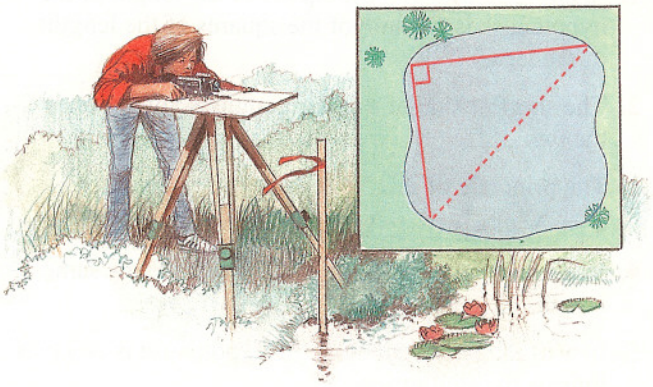
23. *Television Sets.* What does it mean to refer to a 51-in. TV set? Such units refer to the diagonal of the screen. A 51-in. TV set has a width of 45 in. What is its height? 24 in.



24. *Television Sets.* A 53-in. TV set has a screen with a height of 28 in. What is its width? (See Exercise 23.) 45 in.

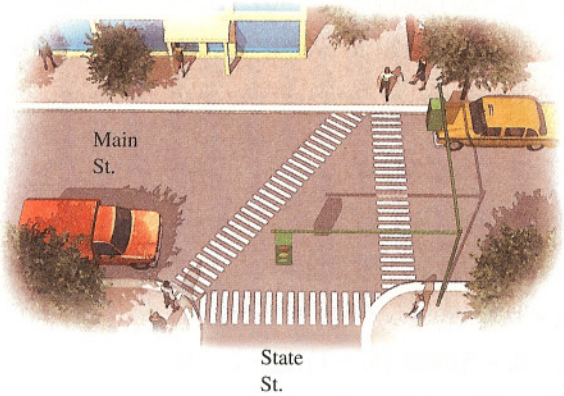
25. *Speaker Placement.* A stereo receiver is in a corner of a 12-ft by 14-ft room. Wire will run under a rug, diagonally, to a speaker in the far corner. If 4 ft of slack is required on each end, how long a piece of wire should be purchased? $\left(\sqrt{340} + 8\right)$ ft; 26.439 ft

26. *Distance over Water.* To determine the width of a pond, a surveyor locates two stakes at either end of the pond and uses instrumentation to place a third stake so that the distance across the pond is the length of a hypotenuse. If the third stake is 90 m from one stake and 70 m from the other, what is the distance across the pond? $\sqrt{13{,}000}$ m; 114.018 m



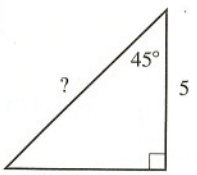
27. *Walking.* Students at Mossway Community College have worn a path that cuts diagonally across the campus "quad." If the quad is actually a rectangle that Angie measured to be 70 paces long and 40 paces wide, how many paces will Angie save by using the diagonal path? $\left(110 - \sqrt{6500}\right)$ paces; 29.377 paces

28. *Crosswalks.* The diagonal crosswalk at the intersection of State St. and Main St. is the hypotenuse of a triangle in which the crosswalks across State St. and Main St. are the legs. If State St. is 28 ft wide and Main St. is 40 ft wide, how much shorter is the distance traveled by pedestrians using the diagonal crosswalk? $(68 - \sqrt{2384})$ ft; 19.174 ft



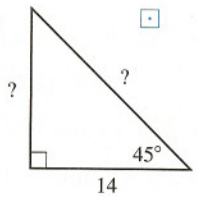
For each triangle, find the missing length(s). Give an exact answer and, where appropriate, an approximation to three decimal places.

29.

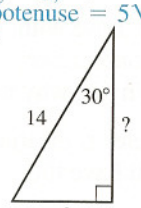


Leg = 5; hypotenuse = $5\sqrt{2} \approx 7.071$

30. ⊡

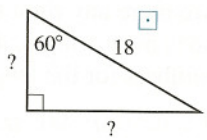


31.

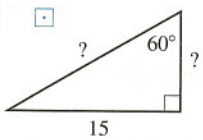


Shorter leg = 7; longer leg = $7\sqrt{3} \approx 12.124$

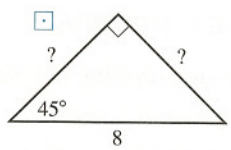
32. ⊡



33. ⊡



34. ⊡



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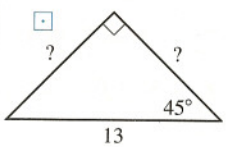
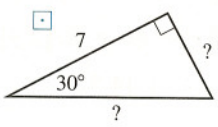


36. ⊡



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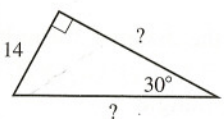
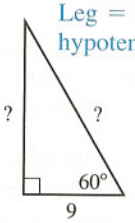


Leg = $14\sqrt{3} \approx 24.249$; hypotenuse = 28

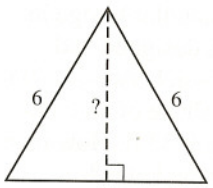
38.



Leg = $9\sqrt{3} \approx 15.588$; hypotenuse = 18

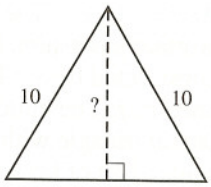
⊡ Answers to Exercises 30, 32–36, 43, and 44 are on p. IA-16.

39.



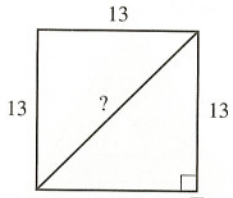
$3\sqrt{3} \approx 5.196$

40.



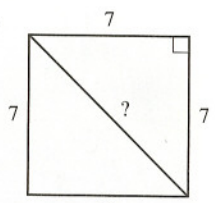
$5\sqrt{3} \approx 8.660$

41.



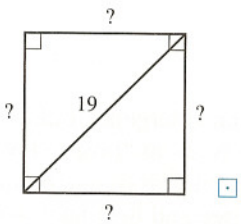
$13\sqrt{2} \approx 18.385$

42.

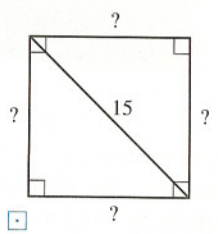


$7\sqrt{2} \approx 9.899$

43.



44.

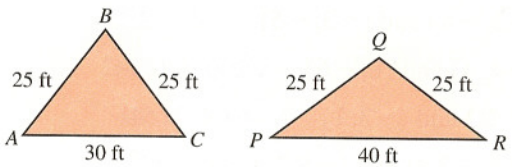


In Exercises 45–50, give an exact answer and, where appropriate, an approximation to three decimal places.

45. *Bridge Expansion.* During the summer heat, a 2-mi bridge expands 2 ft in length. If we assume that the bulge occurs straight up the middle, how high is the bulge? (The answer may surprise you. Most bridges have expansion spaces to avoid such buckling.) $\sqrt{10{,}561}$ ft $\approx$ 102.767 ft

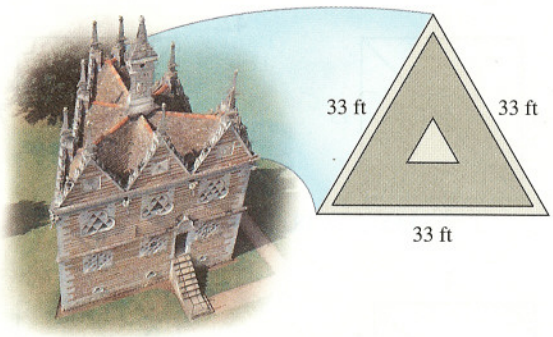
46. Triangle ABC has sides of lengths 25 ft, 25 ft, and 30 ft. Triangle PQR has sides of lengths 25 ft, 25 ft, and 40 ft. Which triangle, if either, has the greater area and by how much?

Neither; they have the same area, 300 ft^2

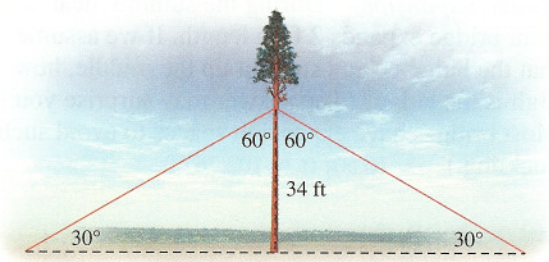


47. *Architecture.* The Rushton Triangular Lodge in Northamptonshire, England, was designed and constructed by Sir Thomas Tresham between 1593 and 1597. The building is in the shape of an equilateral triangle with walls of length 33 ft. How many square feet of land is covered by the lodge? Source: The Internet Encyclopedia of Science

$\frac{1089}{4}\sqrt{3}\ \text{ft}^2 \approx 471.551\ \text{ft}^2$



48. *Antenna Length.* As part of an emergency radio communication station, Rik sets up an "Inverted-V" antenna. He stretches a copper wire from one point on the ground to a point on a tree and then back down to the ground, forming two 30°–60°–90° triangles. If the wire is fastened to the tree 34 ft above the ground, how long is the copper wire? 136 ft



49. Find all points on the y-axis of a Cartesian coordinate system that are 5 units from the point $(3, 0)$. $(0, -4), (0, 4)$

50. Find all points on the x-axis of a Cartesian coordinate system that are 5 units from the point $(0, 4)$. $(-3, 0), (3, 0)$

Find the distance between each pair of points. Where appropriate, find an approximation to three decimal places.

51. $(4, 5)$ and $(7, 1)$ 5

52. $(0, 8)$ and $(6, 0)$ 10

53. $(0, -5)$ and $(1, -2)$ $\sqrt{10} \approx 3.162$

54. $(-1, -4)$ and $(-3, -5)$ $\sqrt{5} \approx 2.236$

55. $(-4, 4)$ and $(6, -6)$ $\sqrt{200} \approx 14.142$

56. $(5, 21)$ and $(-3, 1)$ $\sqrt{464} \approx 21.541$

Aha! **57.** $(8.6, -3.4)$ and $(-9.2, -3.4)$ 17.8

58. $(5.9, 2)$ and $(3.7, -7.7)$ $\sqrt{98.93} \approx 9.946$

59. $\left(\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{5}{6}, -\frac{1}{6}\right)$ $\frac{\sqrt{13}}{6} \approx 0.601$

60. $\left(\frac{5}{7}, \frac{1}{14}\right)$ and $\left(\frac{1}{7}, \frac{11}{14}\right)$ $\frac{\sqrt{41}}{7} \approx 0.915$

61. $\left(-\sqrt{6}, \sqrt{6}\right)$ and $(0, 0)$ $\sqrt{12} \approx 3.464$

62. $\left(\sqrt{5}, -\sqrt{3}\right)$ and $(0, 0)$ $\sqrt{8} \approx 2.828$

63. $(-1, -30)$ and $(-2, -40)$ $\sqrt{101} \approx 10.050$

64. $(0.5, 100)$ and $(1.5, -100)$ $\sqrt{40{,}001} \approx 200.003$

Find the midpoint of each segment with the given endpoints.

65. $(-2, 5)$ and $(8, 3)$ $(3, 4)$

66. $(1, 4)$ and $(9, -6)$ $(5, -1)$

67. $(2, -1)$ and $(5, 8)$ $\left(\frac{7}{2}, \frac{7}{2}\right)$

68. $(-1, 2)$ and $(1, -3)$ $\left(0, -\frac{1}{2}\right)$

69. $(-8, -5)$ and $(6, -1)$ $(-1, -3)$

70. $(8, -2)$ and $(-3, 4)$ $\left(\frac{5}{2}, 1\right)$

71. $(-3.4, 8.1)$ and $(4.8, -8.1)$ $(0.7, 0)$

72. $(4.1, 6.9)$ and $(5.2, -8.9)$ $(4.65, -1)$

73. $\left(\frac{1}{6}, -\frac{3}{4}\right)$ and $\left(-\frac{1}{3}, \frac{5}{6}\right)$ $\left(-\frac{1}{12}, \frac{1}{24}\right)$

74. $\left(-\frac{4}{5}, -\frac{2}{3}\right)$ and $\left(\frac{1}{8}, \frac{3}{4}\right)$ $\left(-\frac{27}{80}, \frac{1}{24}\right)$

75. $\left(\sqrt{2}, -1\right)$ and $\left(\sqrt{3}, 4\right)$ $\left(\frac{\sqrt{2}+\sqrt{3}}{2}, \frac{3}{2}\right)$

76. $\left(9, 2\sqrt{3}\right)$ and $\left(-4, 5\sqrt{3}\right)$ $\left(\frac{5}{2}, \frac{7\sqrt{3}}{2}\right)$

TW **77.** Are there any right triangles, other than those with sides measuring 3, 4, and 5, that have consecutive numbers for the lengths of the sides? Why or why not?

TW **78.** If a 30°–60°–90° triangle and an isosceles right triangle have the same perimeter, which will have the greater area? Why?

SKILL REVIEW

Review graphing (Sections 2.2, 2.3, and 4.5).

Graph on a plane.

79. $y = 2x - 3$ [2.2] ⊡

80. $y < x$ [4.5] ⊡

81. $8x - 4y = 8$ [2.3] ⊡

82. $2y - 1 = 7$ [2.3] ⊡

83. $x \geq 1$ [4.5] ⊡

84. $x - 5 = 6 - 2y$ [2.2] ⊡

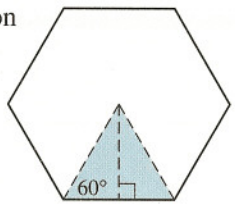
SYNTHESIS

TW **85.** Describe a procedure that uses the distance formula to determine whether three points, (x_1, y_1), (x_2, y_2), and (x_3, y_3), are vertices of a right triangle.

TW **86.** Outline a procedure that uses the distance formula to determine whether three points, (x_1, y_1), (x_2, y_2), and (x_3, y_3), are collinear (lie on the same line).

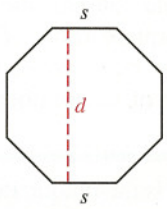
⊡ Answers to Exercises 79–84 are on p. IA-16.

87. The perimeter of a regular hexagon is 72 cm. Determine the area of the shaded region shown.
$36\sqrt{3}$ cm^2; 62.354 cm^2

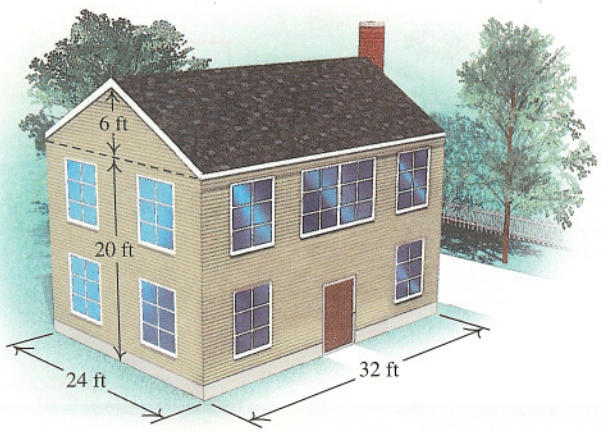


88. If the perimeter of a regular hexagon is 120 ft, what is its area? (*Hint*: See Exercise 87.) $600\sqrt{3}$ ft^2; 1039.230 ft^2

89. Each side of a regular octagon has length s. Find a formula for the distance d between the parallel sides of the octagon. $d = s + s\sqrt{2}$



90. *Roofing.* Kit's home, which is 24 ft wide and 32 ft long, needs a new roof. By counting clapboards that are 4 in. apart, Kit determines that the peak of the roof is 6 ft higher than the sides. If one packet of shingles covers 100 ft^2, how many packets will the job require? 9 packets

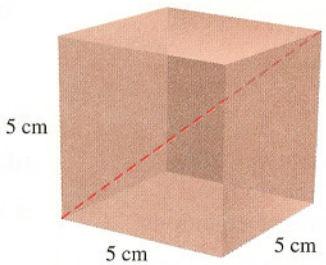


91. *Painting.* (Refer to Exercise 90.) A gallon of Benjamin Moore® exterior acrylic paint covers 450–500 ft^2. If Kit's house has dimensions as shown above, how many gallons of paint should be bought to paint the house? What assumption(s) is made in your answer? 5 gal. The total area of the doors and windows is 134 ft^2 or more.

92. *Contracting.* Oxford Builders has an extension cord on their generator that permits them to work, with electricity, anywhere in a circular area of 3850 ft^2. Find the dimensions of the largest square room they could work on without having to relocate the generator to reach each corner of the floor plan. 49.5 ft by 49.5 ft

93. *Contracting.* Cleary Construction has a hose attached to their insulation blower that permits them to work, with electricity, anywhere in a circular area of 6160 ft^2. Find the dimensions of the largest square room with 12-ft ceilings in which they could reach all corners with the hose while leaving the blower centrally located. Assume that the blower sits on the floor. 60.28 ft by 60.28 ft

94. A cube measures 5 cm on each side. How long is the diagonal that connects two opposite corners of the cube? Give an exact answer. $\sqrt{75}$ cm



95. Prove the midpoint formula by showing that

i) the distance from (x_1, y_1) to

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

equals the distance from (x_2, y_2) to

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right);$$

and

ii) the points

$$(x_1, y_1), \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right),$$

and

$$(x_2, y_2)$$

lie on the same line (see Exercise 86). ⊡

Try Exercise Answers: Section 7.7

15. $\sqrt{19}$ in.; 4.359 in. **19.** 250 ft
29. Leg = 5; hypotenuse = $5\sqrt{2} \approx 7.071$
33. Leg = $5\sqrt{3} \approx 8.660$; hypotenuse = $10\sqrt{3} \approx 17.321$
35. Both legs = $\frac{13\sqrt{2}}{2} \approx 9.192$
37. Leg = $14\sqrt{3} \approx 24.249$; hypotenuse = 28
51. 5 **65.** (3, 4)

⊡ Answer to Exercise 95 is on p. IA-16.